



GCE

Further Mathematics B MEI

Y436/01: Further pure with technology

A Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Y436/01

Mark Scheme

June 2023

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

2. Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

- a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

Y436/01

Mark Scheme

June 2023

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

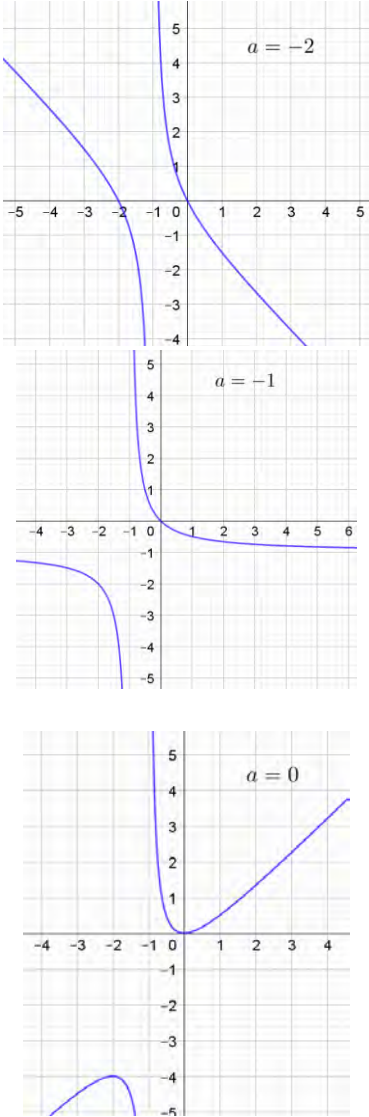
- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
- Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
 - When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
- NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads “3 s.f”
- Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.
- g Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” and “Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Y436/01

Mark Scheme

June 2023

1	(a)	(i)	Answer	Marks	AOs	Guidance
			 <p>The answer section contains three separate coordinate grids, each showing a cubic function curve. The first grid is labeled $a = -2$ and shows a curve that is strictly decreasing, passing through the origin, with a vertical asymptote at $x = -1$. The second grid is labeled $a = -1$ and shows a curve that is strictly increasing, passing through the origin, with a vertical asymptote at $x = -1$. The third grid is labeled $a = 0$ and shows a cubic curve that is strictly increasing, passing through the origin, with a vertical asymptote at $x = -1$ and a local maximum in the third quadrant.</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>1.1</p> <p>1.1</p> <p>1.1</p>	<p>Shape including near asymptotes; position of curves in quadrants relative to axes; points on axes clearly indicated, asymptote (no need sketch the asymptotes themselves).</p> <p>Shape including near asymptotes; position of curves in quadrants relative to axes; points on axes clearly indicated, asymptote (no need to sketch the asymptotes themselves).</p> <p>Shape including near asymptotes; position relative to axes, quadrants; passes through origin, asymptote (no need to sketch the asymptotes themselves). Position of maximum.</p>

Y436/01

Mark Scheme

June 2023

Question		Answer	Marks	AOs	Guidance
(a)	(ii)	The curves all have a vertical asymptote, the line $x = -1$.	B1 [1]	1.2	Any reasonable common feature acceptable. Other acceptable responses include unbounded and pass through the origin.
(a)	(iii)	The curve when $a = 0$ has stationary points whereas there are none in the case of $a = -2$ and $a = -1$.	B1 [1]	1.2	Any reasonable distinguishing feature acceptable. For example, the curve crossing the x -axis for $a = -2$ and $a = -1$ but is tangent to x -axis when $a = 0$.
(b)	(i)	$y = ax + \frac{x^2}{1+x} = \frac{(a+1)x^2 + ax}{1+x}$ $= \frac{(1+x)((a+1)x-1)+1}{1+x} = (a+1)x - 1 + \frac{1}{1+x}$ <p>As $x \rightarrow \pm\infty, \frac{1}{x+1} \rightarrow 0$.</p> <p>Therefore, the oblique asymptote is $y = (a+1)x - 1$.</p>	M1 M1 B1 [3]	1.1a 1.1 1.1	This can be implied by the correct answer. To be implied the answer must include a statement that the final equation is the oblique asymptote. If “oblique asymptote” statement is missing, must see at least one correct limit taken e.g. $x \rightarrow \infty$.

Y436/01

Mark Scheme

June 2023

Question		Answer	Marks	AOs	Guidance
	(b)	(ii)			
		<p>The line between the two points is</p> $y = \frac{ab^2 + b^2 - a}{b^2 - 1}x - \frac{b^2}{b^2 - 1}.$ <p>The point where this crosses the y-axis is</p> $\left(0, -\frac{b^2}{b^2 - 1}\right)$ <p>This is independent of a.</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>1.1a</p> <p>1.1</p> <p>2.2a</p>	<p>Line can be obtained using CAS. Can formulate with $x = 0$ provided working shows reasoning is due to finding the point on y-axis.</p> <p>Accept consideration of the midpoint of the line between the two points.</p> <p>Accept statement of the y-coordinate of point.</p>

Y436/01

Mark Scheme

June 2023

Question	Answer	Marks	AOs		Guidance
(b) (iii)	$y = ax + \frac{x^2}{1+x} = \frac{x((a+1)x+a)}{1+x} \quad (*)$ <p>If $a \geq 0$ and $x \geq 0$ then $(a+1)x+a \geq 0$. If $a \leq -1$ and $x \geq 0$ then $(a+1)x+a < 0$.</p> <p>If $-1 < a < 0$ then</p> $(a+1)x+a \leq 0 \text{ when } 0 \leq x \leq \frac{-a}{1+a}$ <p>and $(a+1)x+a > 0$ when $x > \frac{-a}{1+a}$</p> <p>Therefore by considering (*), $y \geq 0$ for all $x \geq 0$ whenever $a \geq 0$ and $y \leq 0$ for all $x \geq 0$ whenever $a \leq -1$ and neither of these statements is true for $-1 < a < 0$.</p> <p>Therefore</p> <ul style="list-style-type: none"> • $y \geq 0$ for all $x \geq 0$ precisely when $a \geq 0$. • $y \leq 0$ for all $x \geq 0$ precisely when $a \leq -1$ 	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>1.1a</p> <p>2.1</p> <p>2.1</p> <p>2.2a</p>	<p>Must see x factored out in preparation for sign argument. Can be implied by subsequent sign argument working.</p>	
	<p>Alternative method</p> $y = ax + \frac{x^2}{1+x} = \frac{x((a+1)x+a)}{1+x}$ <p>Roots are $x = 0$ and $x = \frac{-a}{a+1}$.</p>	<p>M1</p>	<p>1.1a</p>		

Y436/01

Mark Scheme

June 2023

Question	Answer	Marks	AOs	Guidance
	<p>If $a \geq 0$, there are no positive roots.</p> <p>If $a \leq -1$, $\frac{-a}{a+1} < 0$ so there are no positive roots.</p> <p>$\frac{-a}{a+1} > 0$ if and only if $-1 < a < 0$.</p> $\frac{dy}{dx} = a + \frac{2x(x+1) - x^2}{(x+1)^2}$ <p>At $x = 0$, $\frac{dy}{dx} = a$.</p> <p>If $a > 0$, $\frac{dy}{dx} = a > 0$. If $a \leq -1$, $\frac{dy}{dx} = a < 0$.</p> <p>At $x = 0$, $y = 0$ and if $a > 0$, $\frac{dy}{dx} = a > 0$, so $y \geq 0$ for all $x \geq 0$ precisely when $a \geq 0$</p> <p>At $x = 0$, $y = 0$ and if $a \leq -1$, $\frac{dy}{dx} = a < 0$, so $y \leq 0$ for all $x \geq 0$ precisely when $a \leq -1$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>2.1</p> <p>2.1</p> <p>2.2a</p> <p>2.2a</p>	<p>Condone use of weak inequalities.</p> <p>Condone lack of precise reasoning with biconditional/if and only if statement.</p> <p>Condone use of weak inequalities</p> $= a + \frac{x(x+2)}{(x+1)^2}$

Y436/01

Mark Scheme

June 2023

Question	Answer	Marks	AOs	Guidance
(c)	<p>By plotting the curve it can be seen that the cusp is at the origin. The existence of a cusp at the origin is fully justified as follows.</p> <p>If $y = \left(\frac{x^2}{1+x}\right)^{1/4}$ then $\frac{dy}{dx} = \frac{x(x+2)}{4(x+1)^2} \left(\frac{x+1}{x^2}\right)^{3/4}$.</p> <p>By CAS, $\lim_{x \rightarrow 0^-} \frac{dy}{dx} = -\infty$ and $\lim_{x \rightarrow 0^+} \frac{dy}{dx} = \infty$.</p> <p>Hence the gradient tends to being infinitely steep (negative on the left and positive on the right) as x tends to zero from above and below.</p> <p>When $x = 0, y = 0$ and so $(0,0)$ is on the curve.</p> <p>The curve has a cusp at $(0,0)$.</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>B1</p> <p>[5]</p>	<p>1.1a</p> <p>1.1</p> <p>2.1</p> <p>1.1a</p> <p>2.2a</p>	<p>Must clearly consider the consequence of limits being $\pm\infty$ from above and below in geometric terms. For example, the tangent(s) is vertical.</p>
2	<p>(a)</p> <p>Suppose $c = b+1$ and substitute into $a^2 + b^2 = c^2$.</p> <p>Simplify to $a^2 = b+c$.</p> <p>Suppose $a^2 = b+c$. Substitute into $a^2 + b^2 = c^2$. Simplify to $b+c+b^2 = c^2$.</p> <p>Factorise and rearrange,</p> <p>$(b+c)(1+b-c) = 0$.</p> <p>Since $b+c \neq 0, c = b+1$.</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>1.1a</p> <p>2.2a</p> <p>1.1</p> <p>2.2a</p>	<p>If argument is present as an “if and only if” must see valid reasoning to support this assertion.</p> <p>Or $c = \frac{1 \pm \sqrt{(2b+1)^2}}{2} = b+1$ or $c = -b$.</p> <p>Since $b, c \geq 0, c = b+1$</p> <p>Accept CAS for finding c.</p>

Y436/01

Mark Scheme

June 2023

Question		Answer	Marks	AOs	Guidance
(b)		<p>Appropriate program structure</p> <p>Loops with correct range Check required condition on a and c with if statement.</p> <p>Fully correct program</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>3.3</p> <p>2.1</p> <p>2.5</p>	<p>Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors.</p> <p>For example, use of loops, if statement(s) to check condition(s) and print final output.</p> <p>Condone one incorrect loop with one wrong value for M1 mark.</p> <p>Example program For a in range(1,501): for c in range(a,1001): if a*a + (c-1)*(c-1)==c*c and c-1!=0: print(a,c-1,c)</p>
(c)		21	<p>B1</p> <p>[1]</p>	1.1a	
(d)		<p>If $b^2 = a + c$ then, by a similar argument to that in (a), $c = a + 1$.</p> <p>Since a, b and c are integers and $c = a + 1$, $0 < a \leq b < c$, this implies that $b = a$.</p> <p>Therefore $2a^2 = a^2 + a^2 = a^2 + b^2 = c^2$. This gives that $\left(\frac{c}{a}\right)^2 = 2$ which is not possible because $\sqrt{2}$ is irrational.</p> <p>Therefore, no such Pythagorean triples exist.</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>2.1</p> <p>2.1</p> <p>3.2a</p>	<p>Award A1 for the derivation of any correct contradiction based on earlier M marks.</p>

Y436/01

Mark Scheme

June 2023

Question		Answer	Marks	AOs		Guidance
		<p>Alternative method 1</p> <p>If $b^2 = a + c$ then, by a similar argument to that in (a), $c = a + 1$. So $b^2 = 2a + 1$.</p> <p>For $a \geq 3$, $b = \sqrt{2a + 1} < a$. Which is a contradiction to $0 < a \leq b < c$.</p> <p>When $a = 1$, $b = \sqrt{3}$ and $a = 2$, $b = \sqrt{5}$. Which is impossible since b is an integer.</p>	<p>M1</p> <p>M1</p> <p>A1</p>	<p>2.1</p> <p>2.1</p> <p>3.2a</p>		
		<p>Alternative method 2</p> <p>If $b^2 = a + c$ then, by a similar argument to that in (a), $c = a + 1$. Since a, b and c are integers and $c = a + 1$ $0 < a \leq b < c$, this implies that $b = a = c - 1$. Since $b^2 = a + c$, $b^2 - 2b - 1 = 0$. This equation has no integer solutions.</p>	<p>M1</p> <p>M1</p> <p>A1</p>	<p>2.1</p> <p>2.1</p> <p>3.2a</p>	<p>Accept any correct method leading to deduction.</p> <p>Accept any correct method leading to deduction, for example CAS to find roots which are not integers.</p>	
3	(a)	<p>By Wilson's theorem we have that $18! \equiv -1 \pmod{19}$ since 19 is prime.</p> <p>$18 \equiv -1 \pmod{19}$ and so $18^2 \equiv 1 \pmod{19}$</p> <p>Multiplying both sides by 18 to get $18 \times 18 \times 17! \equiv (-1) \times (-1) \pmod{19}$. giving the stated result since $18^2 \equiv 1 \pmod{19}$</p>	<p>M1</p> <p>A1</p>	<p>1.1a</p> <p>2.2a</p>	<p>Could be implied.</p>	
		Alternative method				

Y436/01

Mark Scheme

June 2023

Question			Answer	Marks	AOs	Guidance
			<p>19 is prime. By Wilson's theorem $18! \equiv -1 \pmod{19} \equiv 18 \pmod{19}$.</p> <p>Since $18! = 18 \times 17!$ and 18 and 19 are coprime $18! \equiv 18 \pmod{19}$ implies $17! \equiv 1 \pmod{19}$.</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>1.1a</p> <p>2.2a</p>	<p>19 is prime can be implied.</p> <p>Coprime requirement can be implied from a statement that 19 is prime to justify the conclusion.</p>

Y436/01

Mark Scheme

June 2023

	(b)	(i)	<p>Appropriate structure program</p> <p>Loops with suitable range with starting value of 2.</p> <p>Check required condition on n with if statement.</p> <p>Fully correct program</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>3.3</p> <p>3.4</p> <p>2.1</p> <p>2.3</p>	<p>Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors.</p> <p>For example, use of loops, if statement(s) to check condition(s) and print final output.</p> <p>Starting value must be such that there are no additional values produced.</p> <p>Example programme def Factorial(n): if n == 1: return 1 else: return(Factorial(n-1)*n)</p> <p>def IsPrime(n): prime = True for i in range(2,n): if n % i == 0: prime = False return prime</p> <p>def WilsonPrime(n): if IsPrime(n): if ((Factorial(n-1))%(n**2))==n**2-1: return True else: return False else: return False</p>	<p>Note that the prime check is not essential here (satisfying the required equation implies that n is prime)</p> <p>Routines for Factorial and Prime functions can be called.</p>
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Y436/01

Mark Scheme

June 2023

Question		Answer	Marks	AOs		Guidance
			[4]		for i in range(2,1001): if WilsonPrime(i): print(i)	
	(ii)	5, 13, 563	B1 [1]	1.1	Need to see 13, 563 and no other values except 5. Condone missing 5.	
	(iii)	<p>Suppose that m is an integer solution to the equation $(p - 1)! + 1 = m^2$ where p is prime.</p> <p>By Wilson's theorem $(p - 1)! + 1$ is a multiple of p.</p> <p>Therefore, m^2 is a multiple of p.</p> <p>Since m^2 is a square each prime occurs in its prime factorisation an even number of times. Therefore m^2 is a multiple of p^2.</p> <p>This means that $(p - 1)! \equiv -1 \pmod{p^2}$ and p is a Wilson prime</p>	M1 M1 A1 [3]	1.1a 2.2a 3.2a	Must include reasoning from the fact that p is prime. Special case. Give SC1 for correct working from m^2 is a multiple of p^2 if insufficient reasoning to award previous M1.	

Y436/01

Mark Scheme

June 2023

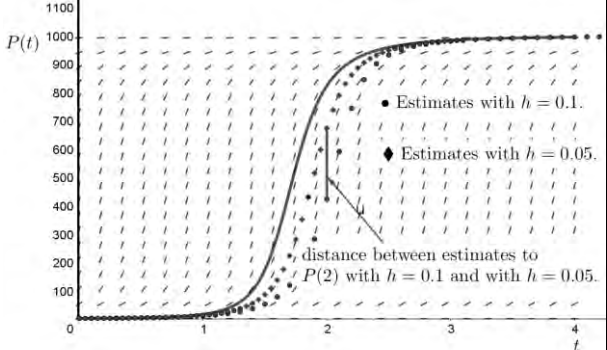
4	(a)	(i)	$\frac{dP}{dt} = rP \left(1 - \frac{P}{K} \right)$ $\frac{dP}{dt} = 0 \Leftrightarrow P = 0 \text{ or } P = K$ <p>Since $r > 0$, by considering the signs of the factors P and $1 - \frac{P}{K}$ for $P > 0$,</p> $\frac{dP}{dt} > 0 \Leftrightarrow 0 < P < K$ $\frac{dP}{dt} < 0 \Leftrightarrow K < P$	B1	1.1a	
				M1	1.1	Consideration of the signs in at least one case.
				A1	1.1	If M0, then SC1 if at least one of final two conditions correct.
				A1	1.1	Condone inclusion of $P < 0$ for final case.
				[4]		
		(ii)	$P(t) = \frac{KP_0 e^{rt}}{K + P_0(e^{rt} - 1)}$	B1	1.1a	By CAS
			[1]			
		(iii)	In both cases $P(t) \rightarrow K$ as $t \rightarrow \infty$	B1	1.1	
			[1]			
		(iv)	$P(t)$ is decreasing if $P_0 > K$ and increasing if $P_0 < K$.	B1	1.2	Allow 'P approaches K from above when $P_0 > K$ and from below when $P_0 < K$.
			[1]			
		(v)	K is the long term population.	B1	3.2a	
			[1]			

Y436/01

Mark Scheme

June 2023

	(b)	(i)	<p>If G1 contains the value of h</p> <p>then</p> <p>A1 contains 0 B1 contains 1 C1 contains $=2*(B1^{1.25})*((1-B1/1000)^{1.5})$ A2 contains $=A1+G\$1$ B2 contains $=B1+G\$1*C1$ C2 contains $=2*(B2^{1.25})*((1-B2/1000)^{1.5})$ and copy down</p>	B1	3.1a	<p>Give reasonable BOD on possible transcription errors and consider correct answers to 4(b)(ii), 4(b)(iii), 4(b)(iv) as evidence of correct formulae in the spreadsheet.</p> <p>Allows for h to be varied.</p>	
				B1	3.1a	Columns for t_n and P_n	
				B1	3.1a	Calculation of $\frac{dP}{dt}$ at each stage.	
						Could be in separate column.	
				B1	2.5	Formulae for t_{n+1} and P_{n+1} . Should include clear indication, for example, “copy down”, for how formulae are generated.	
				[4]			
		(ii)	<p>Approximations using $h = 0.1$ to 7 d.p are</p> <p>$P(1) \approx 9.5679218$ $P(2) \approx 421.7281066$ $P(3) \approx 979.5970106$</p>	B1	1.1	Need all correct to at least 3 s.f.	
				[1]			
		(iii)	<p>Approximations using $h = 0.05$ to 7 d.p are</p> <p>$P(1) \approx 11.7740227$ $P(2) \approx 672.3104086$ $P(3) \approx 981.0771079$</p>	B1	1.1	Need all correct to at least 3 s.f.	
				[1]			
		(iv)	<p>There is an increase in the approximations when using $h = 0.05$ compared to $h = 0.1$.</p>	M1	1.1	The gradient increases (between $t = 1$ and $t = 2$).	A suitable annotated diagram specific to this case could receive M1 A1.

		<p>A smaller h (generally) gives a better estimate. Therefore, values are likely to be underestimates.</p>	<p>A1</p> <p>3.2b</p> <p>[2]</p>	<p>Tangent to curve is below the curve for $t \leq 2$. Hence values are likely to be underestimates (including for $t = 3$ due to the cumulative effect of continually underestimating for $t \leq 2$). o.e.</p>	
	<p>(v)</p>	 <p>There is a large difference in the two approximations for $P(2)$ (compared to $P(1)$ and $P(3)$ approximations).</p> <p>As can be seen from the tangent field, or as in (iv), for lower values of t_n iterations of the Euler method provide underestimates which results in the particularly steep part of the approximation curve being pushed to the right.</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>3.2a</p> <p>3.5a</p> <p>Comparison of relative accuracy of the approximations in (ii) and (iii). Or compare both $P(2)$ approximations to value(s) read from the solution in diagram.</p> <p>Must include the consequence of steep gradient, such as inaccuracies in approximation to gradient of solution curve generate larger differences in approximated P values.</p>	<p>Some quantification of accuracy stronger than that given in (iv).</p>

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